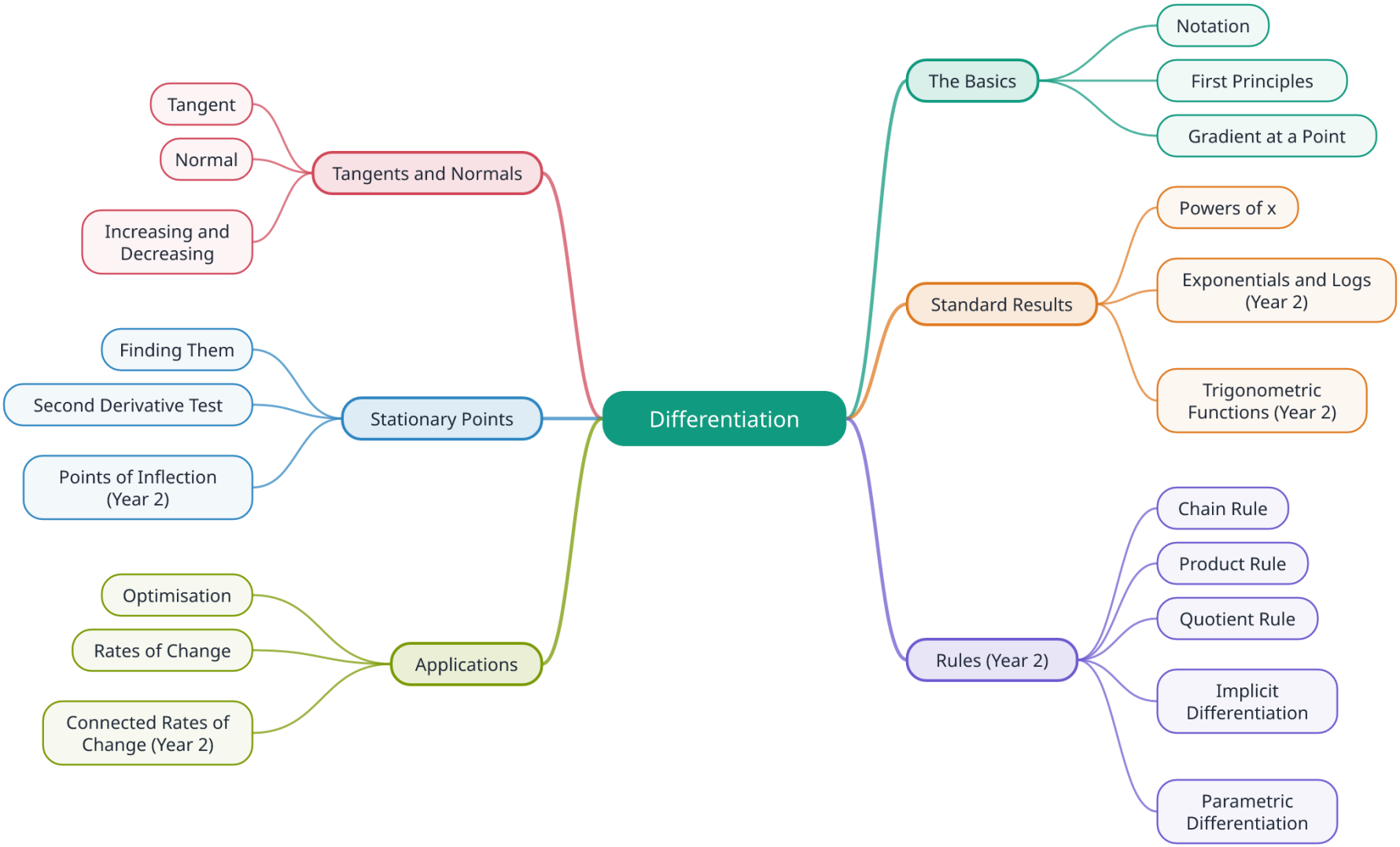
Differentiation Mind Map (A Level Maths)

A level · Mathematics · mindmapi.ai/mind-maps/differentiation-a-level



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Differentiation mind map explained

The Basics

Differentiation finds the gradient of a curve, which is the rate of change of y with respect to x.

Notation

dy/dx or f'(x) is the first derivative; d^2y/dx^2 or f''(x) is the second derivative.

First Principles

f'(x) is the limit, as h tends to 0, of (f(x + h) - f(x)) / h: the gradient of a chord as it shrinks to a tangent. You may be asked to prove simple results this way, such as the derivative of x^2.

Gradient at a Point

Substitute the x-coordinate into dy/dx to find the gradient of the curve there.

Standard Results

Learn these so you can apply them without thinking.

Powers of x

If y = ax^n, then dy/dx = anx^(n-1). Rewrite roots and fractions as powers first, for example 1/x = x^(-1).

Exponentials and Logs (Year 2)

e^(kx) differentiates to ke^(kx). ln x differentiates to 1/x. a^x differentiates to a^x ln a.

Trigonometric Functions (Year 2)

sin kx -> k cos kx; cos kx -> -k sin kx; tan kx -> k $\sec^2 kx$. Angles must be in radians.

Rules (Year 2)

For functions combined in different ways.

Chain Rule

For a function of a function: dy/dx = dy/du × du/dx. For example, (3x + 1)^5 differentiates to 15(3x + 1)^4.

Product Rule

For y = uv: dy/dx = u dv/dx + v du/dx.

Quotient Rule

For y = u/v: dy/dx = (v du/dx - u dv/dx) / v^2.

Implicit Differentiation

Differentiate every term with respect to x. A y term picks up a factor of dy/dx, for example y^2 becomes 2y dy/dx. Then rearrange for dy/dx.

Parametric Differentiation

If x and y are both given in terms of t: dy/dx = (dy/dt) / (dx/dt).

Tangents and Normals

Straight lines that touch or cross the curve at a point.

Tangent

Find the gradient m at the point (x1, y1), then use y - y1 = m(x - x1).

Normal

Perpendicular to the tangent, so its gradient is -1/m.

Increasing and Decreasing

A function is increasing where dy/dx > 0 and decreasing where dy/dx < 0.

Stationary Points

Points where the gradient is zero.

Finding Them

Solve dy/dx = 0 to find the x-coordinates, then substitute into y to find the points.

Second Derivative Test

If $d^2y/dx^2 > 0$ it is a minimum; if $d^2y/dx^2 < 0$ it is a maximum. If it equals 0 the test fails: check the gradient on either side instead.

Points of Inflection (Year 2)

Where the second derivative changes sign. A curve is convex where f''(x) > 0 and concave where f''(x) < 0.

Applications

Using derivatives to solve problems.

Optimisation

Write the quantity to maximise or minimise, such as an area or volume, in terms of one variable, differentiate, set to zero and solve. Check whether it is a maximum or minimum.

Rates of Change

dy/dx measures how fast y changes as x changes, for example velocity as the derivative of displacement with respect to time.

Connected Rates of Change (Year 2)

Link rates with the chain rule, for example dV/dt = dV/dr × dr/dt for a growing sphere.

How to revise with this mind map

1. Learn the standard results until they are instant. Most differentiation questions lose marks on a slipped power or sign, not on the method.

2. Recognise which rule to use: a function of a function needs the chain rule, a product needs the product rule, a fraction needs the quotient rule (or the product rule with a negative power).

3. Always rewrite first. Turn roots into fractional powers and 1/x terms into negative powers before differentiating.

4. For stationary points, show every step: dy/dx = 0, solve, find y, then justify the nature with the second derivative. Method marks depend on the working.

5. Make it yours. Build your own differentiation map in Mindmapi from your notes, and ask the AI about any rule you are unsure of. The answer is saved into that branch's notes for next time.